Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, February 2016
(2013 Scheme)
13.401 : ENGINEERING MATHEMATICS – III (E)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Find and sketch the image of the line $x = 2$ under the transformation $w = e^z$.

2. Evaluate $\int_C (2x - 3y)dz$ where C is the curve joining 1 to $2 + 7i$, along $y + 1 = x^3$.

3. Solve the LPP: Maximize $z = 3x_1 + 2x_2$

   subject to the conditions :

   $x_1 + x_2 \leq 4$

   $x_1 - x_2 \leq 2$

   $x_1 , x_2 \geq 0$

4. Define a subspace. Test if the set $W = \{(x, y, z) \in \mathbb{R}^3 : x^2 = y^2\}$ is a subspace of $\mathbb{R}^3$.

5. Find the possible values of a and b such that {(1, 2, a, 3), (1, b, b, 3)} forms a basis for a two dimensional subspace of $\mathbb{R}^4$.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – 1

6. a) Test the differentiability of the function f(z) at z = 0 if

$$f(z) = \begin{cases} \dfrac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0 , & z = 0 \end{cases}$$

P.T.O.

b) Find the imaginary part v of the analytic function $f(z) = u + iv$ if $u = e^x (x \cos y - y \sin y)$. Show that the v obtained is harmonic.

c) Show that if both $f = u + iv$ and $\bar{f} = u - iv$ are analytic functions, then f must be a constant function.

7. a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the origin.

b) Find all points at which the function $f(z) = z(z^2 - 20)$ fails to be conformal.

c) Find the image of the line $y - x + 1 = 0$ under the transformation $f(z) = \frac{1}{z}$.

Module – 2

8. a) Evaluate $\int_C \frac{\bar{z}}{\pi i} dz$ where C is the upper half of the unit circle $|z| = 1$ from 1 to –1.

b) Show that $\int_0^{2\pi} \frac{\sin\theta}{5 + 4\cos\theta} d\theta = 0$.

c) Use Cauchy's integral formula to evaluate $\oint_{|z|=1} \frac{3z+1}{(2z^2-1)} dz$.

9. a) Find the Laurent series expansion of $f(z) = \frac{z+3}{z^2+z-2}$ in $1 < |z| < 2$.

b) Use Residue theorem to evaluate $\oint_{|z|=2} \frac{e^{2z}}{(z+1)^4} dz$.

c) Evaluate $\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta}$ using contour integration.

Module – 3

10. a) Solve the LPP: Minimize $z = x_1 - 3x_2 - 2x_3$

subject to

$$3x_1 - x_2 - 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

b) Use Big-M method to solve the LPP: Minimize $z = 2x_1 + x_2$

subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 4$$
$$x_1, x_2 \ge 0$$

11. a) Solve the LPP by simplex method: Minimize $z = 6x_1 + 5x_2$

subject to

$$2x_1 + x_2 \ge 80$$
$$x_1 + 2x_2 \ge 60$$
$$x_1, x_2 \ge 0$$

b) Solve the LPP: Maximize $z = 3x_1 - x_2$

subject to

$$2x_1 + x_2 \ge 2$$
$$x_1 + 3x_2 \le 2$$
$$x_2 \le 4$$
$$x_1, x_2 \ge 0$$

Module – 4

12. a) Find the value of a if the polynomials $1 - 2x + 3x^2$, $2 - 4x^2$, $3 - 5x + ax^2$ are linearly dependent.

b) Show that $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$T(x, y, z) = (x - y, x + y, z)$ is a linear transformation.

c) Find the null space of the linear transformation T given in (b). Also find the nullity and range space of this transformation T.

13. a) Find a vector from the span {(1, 1, 2), (1, 0, 1)} which is orthogonal to the vector (0, 1, 1) in $\mathbb{R}^3$.

b) Find an orthonormal basis from {(1, 1, 1), (0, 1, 1), (0, 0, 1)}.

c) Test if {(1, 1, 2), (1, 0, 1), (0, 1, 1)} is linearly independent in $\mathbb{R}^3$. Is this set orthogonal ? Justify your answer.
